Summarization of Optimization Methods

Qinghua Ding

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1 Nonlinear Optimization

- Gradient Method. $x_{k+1} = x_k - h_k f'(x_k)$
- Newton Method. $x_{k+1} = x_k - [f''(x_k)]^{-1} f'(x_k)$
- Damped Newton Method(divergence). $x_{k+1} = x_k - h_k [f''(x_k)]^{-1} f'(x_k)$
- Quasi-Newton Method(degenerate). $x_{k+1} = x_k - h_k H_k f'(x_k)$
  $H_{k+1}(f'(x_{k+1}) - f'(x_k)) = x_{k+1} - x_k$
- Conjugate Gradient. $x_k = \arg\min\{f(x) | x \in x_0 + L_k\}$
- Penalty Function(constrained). $x_{k+1} = \arg\min_{x \in R}\{f_0(x) + t_k \Phi(x)\}$
- Barrier Function(constrained). $x_{k+1} = \arg\min_{x \in Q}\{f_0(x) + \frac{F(x)}{t_k}\}$
  Require Slater condition: $\exists x, \forall i, f_i(x) < 0$

2 Smooth Convex Optimization

- Gradient Descent. $O(\frac{1}{\epsilon})$ for $F_L^{1,1}(R^n)$
  $O(ln\frac{1}{\epsilon})$ for $S_{\mu,L}^{1,1}(R^n)$
- Optimal Methods. $O(ln\frac{1}{\epsilon})$ for $S_{\mu,L}^{1,1}(R^n)$
  Based on Global Estimate Sequence
- Gradient Mapping(minmax). $g_Q(x;\gamma) = \gamma(x - x_Q(x;\gamma))$
- Sequential Quadratic Optimization(constrained).

3 Nonsmooth Convex Optimization

- Basic Ideas. Subgradient; Separation Theorem
- Subgradient. $O(\frac{1}{\epsilon^2})$
- Cutting Plane Method with Center Gravity. $O(nln\frac{1}{\epsilon})$
- Ellipsoid Method. $O(n^2 ln\frac{1}{\epsilon})$
- Kelly Method. Unstable for Practice
- Level Method. $\Omega(\frac{1}{\epsilon^2})$
- Optimal Method. $O(nln\frac{1}{\epsilon})$

4 Extenions of Convex Optimization

- Cubic Regularization
- Trust Region Method